LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – MATHEMATICS

THIRD SEMESTER – NOVEMBER 2018

MT 3501 – ALGEBRA, CALCULUS AND VECTOR ANALYSIS

Date: 29-10-2018 Dept. No. Max. : 100 Marks

Time: 01:00-04:00

PART – A

Answer ALL questions. **(10 × 2 = 20)**

1. Evaluate $\int_0^1\int_0^1\int_0^a xdxdydz$
2. Find $\frac{\partial(u,v)}{\partial(x,y)}$ when $u = x^2 - y^2; v = x^2 + y.^2$
3. Eliminate the arbitrary constants from $z=\left(x^2+a\right)\left(y^2+b\right)$.
4. Solve $p + q = \sin x + \sin y$.
5. If $\varphi(x,y,z)=x^2y+y^2x+z^2$, find $\nabla\varphi$ at $(1,1,1)$.
6. Prove that div $\vec{r}=3$, where $\vec{r}$ is the position vector.
7. Find L (Sin2t).
8. Compute $L^{-1}\left[\frac{1}{s-3}+\frac{1}{s-4}\right]$
9. Find the sum of all divisors of 360.
10. State Fermat's theorem.

PART – B

Answer any FIVE questions **(5 × 8 = 40)**

11. Change the order of integration and evaluate the integral $\int_0^\infty\int_x^\infty \frac{e^{-y}}{y}dxdy$.
12. Express $\int_0^1 x^m\left(1-x^n\right)^p dx$ in terms of Gamma functions and evaluate the integral $\int_0^1 x^5\left(1-x^3\right)^{10} dx$.
13. Solve $p^2 + pq = z^2$.
14. Show that the vector $2xy\hat{i}+(x^2+2yz)\hat{j}+(y^2+1)\hat{k}$ is irrotational.
15. Solve $p^2 + q^2 = z^2(x^2 + y^2)$.
16. Find the value of $L\left[\frac{cos3t - cos2t}{t}\right]$.
17. Evaluate $L^{-1}\left[\frac{1}{(S^2+a^2)^2}\right]$.
18. Show that $18! + 1$ is divisible by 437.

PART – C

Answer any THREE questions. **(2 × 20 =40)**

19. (a) Evaluate $\iint xydxdy$ taken over the positive quadrant of the circle $x^2+y^2=a^2$.

(b) Prove that $\beta(m,n) = \frac{\lceil m \lceil n}{\lceil m+n}$. **(10+10)**

20. (a) Find the general solution of $(y+z)p + (z+x)q = x+y$.

(b) (i) Solve: $p^2 + q^2 = npq$.(ii) Solve: $z^4q^2 - z^2p = 1$. **(10+10)**

21. Solve the equation $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = sint$ given that $y = \frac{dy}{dt} = 0$ when $t = 0$.

22. (a) Verify Green's theorem in the *XY* plane for $\int_C (xy + y^2)dx + x^2dy$ where *C* is the closed curve in the region bounded by $y = x$; $y = x^2$.

(b) State and prove Wilson's theorem. **(10**

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